THE EARLY CASSIRER AND HEGEL'S «CONCRETE UNIVERSAL» IN THE PHILOSOPHY OF MATHEMATICS

by Luigi Laino*

Abstract. *In this paper, I aim to reconstruct the train of thoughts that pushed Cassirer to incorporate «concrete universality» in his philosophy of mathematics. To accomplish this task, I will first concentrate on Hegel's claims regarding the mathematical infinite in his* Science of Logic, *as well as the relationship of true infinity to the concept of function and the «concrete universal». I will then deal with the most relevant intermediate stages from Hegel to Cassirer. In particular, I will dwell upon Drobisch's account of concrete universality in logic and mathematics and Cohen's book on calculus. Finally, I will evaluate Cassirer's early production to explain why his philosophy of mathematics is sympathetic with that of Hegel and how Cassirer radicalises Hegel's position.*

Keywords. *Hegel; Cassirer; Philosophy of Mathematics; Concrete Universal; Functions*

1. *Introduction*

According to a well-established tradition in the field of Cassirer's studies that starts with Hendel and Verene, a crucial aspect of Cassirer's thought is his relationship with Hegelian philosophy[1].

* Università degli Studi di Napoli Federico II

[1] C.W. Hendel, *Introduction*, in E. Cassirer, *The Philosophy of Symbolic Forms, Vol. 1: Language*, trans. by R. Manheim, New Haven-London, Yale University Press, 1955, pp. 1-65; D.P. Verene, *Kant, Hegel, and Cassirer: The Origins of the Philosophy of Symbolic Forms*, «Journal of the History of Ideas», XXX (1), 1969, pp. 33-46. Linke already noted that the Marburg School «is in many respects reminiscent of Hegel» (P.F. Linke, *Logic and Phenomenology*, in *Philosophy Today. Essays on Recent Developments in the Field of Philosophy*, ed. by E.L. Schaub, London, Open Court, 1928, pp. 359-392, in particular pp. 367-369).

Hendel even dared to show that Hegel's philosophy was exploited by Cassirer for his reform of Kantianism. Significantly, the concept of the «concrete universal» would play a pivotal role in this remake. Through this notion Cassirer would identify the «theoretical» and the «factual», as well as he would express the «truth of the particulars» in terms of the «universal»[2].

Recently, scholars have suggested that Hegel's influence is apical in the philosophy of mathematics[3], but the topic has remained mostly unexplored. With this paper, I accordingly hope to expand on this. As to its structure, I will devote the second section to explaining how Hegel interpreted the true infinite in mathematics and how this relates to the concrete universal. In the third section, I will deal with the historical path that went from Hegel to Cassirer, dwelling upon Drobisch's and Cohen's works. In the fourth section, I will clarify why Cassirer believes that concrete universality is actualised in mathematics. The fifth section will contain the concluding remarks.

2. *Hegel: Infinity, the Concept of Function and the Concrete Universal*

Considering the importance of the notion of infinity in Hegel's system, and the purpose of overcoming the antithesis between finiteness and infinity[4], it is no wonder that Hegel's philosophy of mathematics can be seen as a quest for the sublation of «bad infinity» (*schlechte Unendlichkeit*) into the true mathematical infinite[5]. In essence, by rephrasing Hegel's position in more common terms, Hegel shed light on the flaws concerning potential infinity.

[2] Hendel, *Introduction*, p. 33.

[3] E. Skidelsky, *Ernst Cassirer. The Last Philosopher of Culture*, Princeton, Princeton University Press, 2008; G.S. Moss, *Ernst Cassirer and the Autonomy of Language*, Lanham *et al.*, Lexington Books, 2015.

[4] R. Bodei, *La civetta e la talpa. Sistema ed epoca in Hegel*, Bologna, il Mulino, 2014, pp. 281-284.

[5] T. Pinkard, *Hegel's Philosophy of Mathematics*, «Philosophy and Phenomenological Research», XLI (4), 1981, pp. 452-464, in particular pp. 461-464.

Hegel's starting point is that the concept of series constitutes an obstacle for grasping true infinity. Indeed, it simply generates finite elements after finite elements and gives rise to the so-called «and-so-on-theory»[6]. Hegel even speaks in this respect of a «*perennierendes Erzeugen*»: «As it is primarily posited, it makes the infinite the goal, which, however, is not reached: it is a perpetuated creation of it, while yet Quantum is never left behind and the infinite never becomes positive and present»[7]. Thus, we have already encountered two features of the true mathematical infinite: first, (i) it must entail the sublation of *quanta*, which are merely quantities that being counted slide into other finite quantities; on the other hand, (ii) the infinite must lose its status of oughtness or beyondness and be grasped as *infinitum actu*.

To understand this point, we may think of ratios and the possibility to express them with infinite sums. Hegels refers to the fraction *2/7*, which can be written as the infinite decimal number *0,285714...* and so as the series *0/1 + 2/10 + 8/100 + 5/1000 + 7/10000 + ...* The infinite decimal number and the series are bad infinities or simply «enumerations» (*Anzahlen*), while the ratio as such is «the *finite expression* of it»[8]. The following remarks made by Hegel are consequently of great importance:

> For the *infinite series* contains bad infinity, since that which the series is designed to express remains as *Ought*; and what it does express is infected with a Beyond that never vanishes, and is *distinct* from what it desired to express. It is infinite, not because of the number of its terms, but because they are incomplete, because the Other, which essentially belongs to them, is

[6] P. Stekeler-Weithofer, *Mathematical Thinking in Hegel's* Science of Logic, «Internationales Jahrbuch des Deutschen Idealismus», III, 2005, pp. 243-260, pp. 243-244.

[7] G.W.F. Hegel, *Gesammelte Werke*, Hamburg, Meiner, 1968 ff., Bd. 21: *Wissenschaft der Logik. Erster Teil: die objektive Logik. Erster Band: die Lehre vom Sein (1832)*, p. 220 (abbreviation: *GW*). Excerpts are cited according to the English translation by W.H. Johnston and L.G. Struthers, *Science of Logic*, 2 voll., London, Allen & Unwin, vol. 1, p. 242. Henceforth, references to the English translations will be bracketed.

[8] *GW*, Bd. 21, p. 243 (263).

> beyond them; that which it really contains [...] is but something finite, in the proper sense, posited as finite – that is as something *which is not what it ought to be*. That, on the contrary, which is called the *finite expression* or *sum* of such a series, has nothing lacking: it contains fully the value which the series only seeks after; the Beyond is recalled from its flight; what it is, and what it ought to be, are not severed, but the same[9].

Hegel then goes on to speak of series that involve incommensurability and says that in this case we cannot represent the ratio as a *quantum*, even if we use a fraction. Clearly, the question is anew that everything that is connected to «the *form* of the series»[10] yields bad infinity in principle. By way of example, Zeno's well-known paradox of the line would constantly give finite *quanta* in such a way that the succession is a false sublation of finiteness. Therefore, if we consider the set *S* {*1/2, 1/4, 4/8, ..., (1/2)*^{n}}, there is no rational number that seems to close it up.

It is thus no surprise that Hegel vehemently attacked the main trend within the so-called metaphysics of calculus of his time, to wit, the idea that nth terms of infinite series, the infinitesimals, are vanishing quantities[11]. And it is no less crucial that, in a time when the definition of limit was unstable[12], Hegel comes close to it:

[9] Ivi, p. 245 (264).

[10] Ivi, p. 246 (265).

[11] The literature on this topic is immense. While Pinkard blamed Russell for not realising that Hegel attacked the metaphysics of calculus, others have upheld that Hegel does not give up the concept of vanished *quanta* (H. Somers-Hall, *Hegel and Deleuze on the Metaphysical Interpretation of the Calculus*, «Continental Philosophy Review», XLII (4), 2010, pp. 555-572). As to the first trend, Stekeler-Weithofer underlines that in Hegel one even finds a sort of Russellian, to wit, Platonist understanding of mathematical concepts, see P. Stekeler-Weithofer, *Hegels Wissenschaft der Logik. Ein dialogischer Kommentar*, Hamburg, Meiner, 2019, pp. 956-958. The second trend is, in any case, a minoritarian position. See R.M. Kaufmann and C. Yeomans, *Hegel on Calculus*, «History of Philosophy Quarterly», XXXIV (4), 2017, pp. 371-389; S. Houlgate, *Quantity and Measure in Hegel's «Science of Logic»*, London *et al*., Bloomsbury, 2022, pp. 209-244.

[12] See U. Bottazzini, *The Higher Calculcus: A History of Real and Complex Analysis from Euler to Weierstrass*, New York, Springer, 1986.

> If $y = f(x)$, then $f(x)$, when y passes over into $y + k$, is to *change* into $f(x) + ph + qh^2 + rh^3 + \ldots$, and thus $k = ph + qh^2 + \ldots$, and $k/h = p + qh + rh^2 + \ldots$ Now if k and h vanish, the second member, except p, also vanishes: p thus is the limit of the ratio of the two increments. Clearly h (as Quantum) is equated with 0, while k/h is nevertheless not supposed to equal $0/0$, but still to remain a ratio. Now the idea of the *limit* is supposed to afford the advantage of averting the implied inconsistency; and p at the same time is supposed to be, not the actual ratio (which would be $= 0/0$), but only the determinate value to which the ratio may *approximate infinitely*, that is, in such a manner that *the difference can become smaller* than any given difference[13].

The tipping point is that this being smaller than any other quantity is not a possibility. Rather, it «shall» be so[14]. Hence, Hegel takes on the attempt at getting rid of vanishing quantities and other misleading concepts (such as 'infinite approximation', 'continuous magnitude' and so on).

In this sense, his program is consistent with but also different from the arithmetisation of analysis initiated by Lagrange and Cauchy among others[15]. Indeed, we may take p as a coefficient

[13] *GW*, Bd. 21, p. 266 (284).

[14] Ivi, p. 266 (285).

[15] Insisting on the concept of series, Lagrange aimed at getting rid of vanishing quanta. He relied on the Taylor formula which gives: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, and showed that the nth derivatives obeys a succession determining each element as follows: $f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$, both singularly and in its dependence on the others. However, since each member can be assumed to be greater than the sum of the following elements, in such a way that 'rests' can be dropped as uninfluential, Lagrange's approach does not enter the realm of quality. See A. Moretto, *Hegel e la «matematica dell'infinito»*, Trento, Verifiche, 1984, pp. 199-201; Stekeler-Weithofer, *Hegels Wissenschaft der Logik*, pp. 948-949. Also, in Hegel's opinion Cauchy did not avoid the reference to the notion of convergence to a limit and accordingly to bad infinity. Although with some limitations, it is Newton who is credited by Hegel for the exact definition of ultimate ratio. See M. Wolff, *Hegel und Cauchy. Eine Untersuchung zur Philosophie und Geschichte der Mathematik*, in *Hegels Philosophie der Natur*, ed. by R. H. Horstmann and M.J. Petry, Klett-Cotta, Stuttgart, 1986, pp. 197-263.

derivable from: $\frac{f(x+i)-f(x)}{i} = p$[16]. *P* is the derivative and can be represented as a *qualitative* and 'ultimate ratio' because it combines variations on the ordinates with those on the abscissae in such a manner that when *x* changes, *y*, to wit, *f(x)* undergoes a mutual variation that equals *P* as *i* approximates to *0*[17]. Hegel highlights that

> [...] So little is the qualitative ratio here lost, that it is precisely the very result of the conversion of finite magnitude into infinite. [...] Thus, for instance, in the *ultimate ratio* the Quanta of abscissae and of ordinates vanish; but the sides of this ratio essentially remain, the one an element of the ordinates, and the other of the abscissae. [...] The difference, no longer being a difference between finite magnitudes, has ceased to be a manifold within itself: it has collapsed into simple intensity, into the determinateness of one qualitative moment of the ratio relatively to the other[18].

Accordingly, the vanishing of magnitudes corresponds to a naïve representation of what happens geometrically. Derivatives are in fact tangent to curves, which means that the equations of the former are lowered by at least one degree as compared to the equations of the latter. This can be inferred from the expression: $dx^n = nx^{n-1}\,dx$, which combines powers of different degrees[19]. In this case, the formula relates magnitudes to each other without assuming that either or both of them flow away. The function should conversely imply the unity of magnitudes through the relation between powers and is an «indivisible symbol»[20].

[16] *GW*, Bd. 21, p. 273 (291).

[17] See footnote n. 15 and Wolff, *Hegel und Cauchy*, pp. 231-243. The identification of the differential quotient with the derivative depends on the fact that $dy = f'(x)\cdot\Delta x$; then, if $\Delta x = dx$, we have $f'(x) = dy/dx$. However, $dy \neq \Delta y$ (Moretto, *Hegel e la «matematica dell'infinito»*, p. 43). Δx and Δy are finite differences between abscissae and ordinates. See also: M. Giovanelli, *Reality and Negation – Kant's Principle of Anticipations of Perceptions*, Dordrecht *et al.*, Springer, 2011, pp. 124-144; Stekeler-Weithofer, *Hegels Wissenschaft der Logik*, p. 940.

[18] *GW*, Bd. 21, pp. 268-269 (286-287).

[19] See Kaufmann and Yeomans, *Hegel on Calculus*, pp. 380-381.

[20] *GW*, Bd. 21, p. 251 (269-270). See Wolff, *Hegel und Cauchy*, p. 226.

An interesting consequence is that linear functions are excluded from the realm of the true infinite. The slope of a straight line is not a true infinite because it is constant. If we differentiate $y = ax$ and $y = ax + b$, we will have $dy/dx = a$[21]. Quite the contrary, if it is true that «*with the first term* the differential is fully found» and that the development of series implies «but the *repetition* of one and the same *ratio*, which alone is aimed at and is *complete* already *in the first term*»[22], it is no less correct to state that only with differential ratios, and thus with derivatives and curves, do we have an identical law that nonetheless generates different elements and that each of them conveys variations.

I suggest that this is the very definition of the true mathematical infinite expressed through the concept of function[23], and I believe that this idea is well-established among Hegel's scholars. To this purpose, I report below three excerpts from Pinkard and Moretto:

> The actual infinite is an *ideality*, it is simply the representation (*something ideal*) of a sequence (a movement, *Bewegung*, in Hegel's terminology) by a rule which shows what would result if the sequence were followed out. The affirmative infinite thus *is* the potential infinite represented by a rule which shows what would happen were the process to be carried through[24].
>
> [...] A convenient definition of the true quantitative infinity according to Hegel is that of a *unitary comprehension of infinite manifoldness*[25].

[21] *GW*, Bd. 21, p. 250 (268-269). See Wolff, *Hegel und Cauchy*, pp. 243-245, pp. 251-252, pp. 256-257. In short, although being a finite term, *P* is not 'fixed' as *c* in either $x = cy$ or $x/y = c$ (see ivi, p. 243). In this way, constants are not parts of functions, which are *stricto sensu* continuous and differentiable everywhere (see ivi, pp. 259-260).

[22] *GW*, Bd. 21, p. 264 (282). See Wolff, *Hegel und Cauchy*, p. 249.

[23] See Wolff, *Hegel und Cauchy*, p. 207.

[24] Pinkard, *Hegel's Philosophy of Mathematics*, p. 462.

[25] Moretto, *Hegel e la «matematica dell'infinito»*, p. 180.

> Hegel believes that the concept of the *true infinite* is contained in mathematical examples in which we take under consideration *all* the infinite elements of a manifold, which comply with certain conditions, to wit, obey a well specified law[26].

Nevertheless, we should realise that Hegel goes a step further if we frame his position in light of the history of mathematics. In fact, we said that Hegel's approach is sympathetic with the arithmetisation of geometry[27], but we cannot fail to acknowledge that this attempt is not utterly realised because Hegel is wary of mathematical symbolism.

Let us begin with two excerpts from Lagrange and Hegel:

> We will designate, in general, through the feature *f* or *F*, posed before a variable, every function of this variable, that is to say, every quantity that depends on this variable and varies with it according to a given law (*suivant une loi donnée*)[28].

> We discover as its object – *of calculus* – equations in which any number of magnitudes [...] are combined into one determinate whole in such a manner that, *firstly*, they have their determinateness in *empirical magnitudes* which are their fixed limits, and, moreover, in the particular kind of union with them and with one another, which, indeed, is the case with equations generally; but, since there is only *one* equation for both magnitudes [...], these equations belong to the class of *indeterminate* equations; – and, *secondly*, that one aspect of them (the determinateness of these magnitudes here being what it is) is that they are, or at least one of them is, present in the equation in a *higher power* than the first. [...] These magnitudes are wholly of the character of such *variable* magnitudes as occur in the problems of *indeterminate* analysis. Their value is indeterminate, but is so in such a manner that when the one gets a perfectly determinate value (a numerical value) from without, then the other too is determined: one is

[26] Id., *Questioni di filosofia della matematica nella «Scienza della logica» di Hegel. «Die Lehre vom Sein» del 1831*, Trento, Verifiche, 1988, p. 21.

[27] Stekeler-Weithofer, *Hegels Wissenschaft der Logik*, p. 695.

[28] J.-L. Lagrange, *Théorie des fonctions analytiques* (1813^2), in *Oeuvres*, Bd. IX, ed. by J.-M. Serret, Paris, Gauthier-Villars, 1881, p. 21.

> a *function* of the other. The categories of variable magnitudes, functions, and the like, are, therefore, merely *formal* for that specific determinateness of magnitude with which we are dealing here [...]; for they are of a generality not yet containing that specific factor which is the one aim of the differential calculus; nor can that factor be thereby explained through analysis[29].

Lagrange's passage is a technical definition of the notion of function which Hegel seems to have recalled in his words. This is no surprise since arguments of that sort circulated since Euler and were later exploited by Cauchy[30]. Nevertheless, dealing with indeterminate systems of equations, Hegel points out that in this case the concept of function is formal[31]. Also, it is possible to extend this consideration[32] and consequently undermining the import of symbols in mathematics. Therefore, despite Hegel's emphasis that in case of powers and series of powers we are vis-à-vis with a «*relation*» (*Beziehung*) that overarches the unfolding of elements[33], the assumption that the concept of function is formal hinders the realisation of concrete universality in mathematics. At the end of the section on the quantitative ratio, symbols and numbers are indeed presented as means of sensibility or of imagination. Hegel writes:

> In so far as expressions of powers are used only as *symbols*, they are unobjectionable, just as much as are numbers and other symbols of concepts, – but also they are as objectionable as all symbolism whatever which attempts to represent pure conceptual or philosophic determinations. [...] The common determinations of force, or substantiality, cause and effect, and others, are themselves too only symbols used to express other relations, like vital and spiritual relations; that is, they are untrue determinations of those relations [...]. If numbers, powers, the mathematical infinite, and the like are to be used not as symbols but

[29] *GW*, Bd. 21, pp. 276-277 (294-295, mod.).

[30] Wolff, *Hegel und Cauchy*, p. 258.

[31] See Moretto, *Hegel e la «matematica dell'infinito»*, pp. 238-243.

[32] Wolff, *Hegel und Cauchy*, pp. 257-260.

[33] *GW*, Bd. 21, pp. 279-280 (297-298).

> as forms for philosophic determinations and hence themselves as philosophic forms, then first of all their philosophic meaning, that is, their conceptual determinateness, must be demonstrated. If this is done, they are superfluous designations: the conceptual determinateness designates itself, and its own is the only correct and fitting designation. The use of these forms is, therefore, nothing but a convenient means of escaping the trouble of seizing, proclaiming, and justifying the conceptual determination[34].

Considering this, it is not by chance that a more appropriate definition of concrete universality is to be found in the section devoted to the apodeictic judgements:

> Subject and predicate correspond to each other and have the same content, and this *content* itself is *concrete* posited *universality* [*die gesetzte* konkrete Allgemeinheit]; for it contains the two moments, the objective universal (or *genus*) and *the individualised entity*. It is here therefore the universal which is *itself*, and continues itself through *its opposite*, and is universal only as *unity* with the latter[35].

To say that a certain thing can be predicated of a general quality, for instance when we state, 'this deed is right', we should presuppose that the copula contains «the relation of the subject to universality»[36]. In practice, the predication works here as a to-be-reached identity for universality is meant to enable the «correspondence» (*Entsprechen*) of the predicate with the subject, so that the negation of the former would discard the identity of the latter[37].

If this be the case, it is no wonder that concrete universality belongs to the world of biology and its logic, and not to mathematics. In fact, if we can see Hegel's philosophy as an attempt at reducing

[34] *GW*, Bd. 21, p. 322 (344, mod.).

[35] *GW*, Bd. 12: *Wissenschaft der Logik. Zweiter Band. Die subjective Logik* (1816), p. 88 (vol. 2, 298).

[36] *Ibidem* (299).

[37] K.J. Harrelson, *Logic and Ontology in Hegel's Theory of Predication*, «European Journal of Philosophy», XXIII (4), 2015, pp. 1259-1280.

the gulf between the universal and the particular[38], it is only «life» that bridges this gap. The long path that leads from the criticism of Kant's subjective teleology to the section on the notion of «kind» bears witness to this point. Despite Hegel's peculiar mixture of arguments, the clearest example concerns the relation between progenitors and their offspring: here the sublation of individual life is wholly accomplished, for the universality of the kind is reproduced painstakingly in the particular beings[39]. So, we cannot say that the universal of a lion does not predicate 'this' specific lion, otherwise the latter would not be a lion. In biological terms, the «generation of individuality» entails its «transcendence» (*Aufheben*), which means that the two are the moments of the «*für sich werdende Allgemeinheit* der Idee»[40].

Of course, I did not want to provide an exhaustive frame of Hegel's biological thinking. This digression should simply explain why Hegel was not content with the mathematical infinite. If it is true that the latter is actualised in a single object, e.g., a point[41], this happens only in an abstract fashion.

However, we have said that Hegel sought to 'purify' mathematics from the physicalist lingo of calculus[42] and that he tried not to consider the differential properly as a magnitude, but as a whole of «syncategorematic moments»[43]. We will see that Cassirer's philosophy of mathematics will radicalise these thoughts considering new mathematical research.

[38] J.N. Findlay, *Hegel's Use of Teleology*, «The Monist», XLVIII (1), 1964, pp. 1-17.

[39] A. Gambarotto, *Vital Forces, Teleology and Organization. Philosophy of Nature and the Rise of Biology in Germany*, Cham, Springer, 2018, pp. 125-127.

[40] I kept the German original for the English translation (vol. 2, p. 415) is mistaken.

[41] Hegel was among one of the few philosophers who understood this: B. Bolzano, *Paradoxien des Unendlichen* (1851), Leipzig, Meiner, 1920, p. 11. We should also bear in mind that his favourite example concerns Spinoza's eccentric circles, a single *object* in which infinity is embedded: *GW*, Bd. 21, pp. 247-249 (266-269).

[42] Stekeler-Weithofer, *Hegels Wissenschaft der Logik*, p. 943.

[43] Ivi, p. 952.

3. *From Hegel to Cassirer*

For reasons of space, I cannot provide an extended reconstruction of the epoch that led from Hegel's death to the dawn and development of neo-Kantianism[44]. Nevertheless, it is worth focussing at least on Drobisch's mention of concrete universality, and Cohen's work on calculus. The first point is crucial since Cassirer brings into play the concrete universal by mentioning Drobisch alongside with Hegel; as to the second matter, I believe that Cohen's ideas may be of service for realising what change of perspective is urged by Cassirer.

Moritz Wilhelm Drobisch (1802-1896) was a polyhedric figure who contributed to many fields of knowledge, but mainly to empiric psychology and the reform of logic[45]. In outline, Drobisch aimed to distinguish logic from psychology by acknowledging that there are «*Naturgesetze des Denkens*» and «*Normalgesetze*» that obey different epistemological schemes. Whereas the former are «regulative» and «*descriptive*» and unfold the way in which we actually think, the latter are «*demonstrative*» and concern «*precepts*» (*Vorschriften*) and «norms» that we use to distinguish absolutely true from false knowledge. Furthermore, logic is compared to mathematics since it rests on «principles» (*Grundsätze*) and «consequences» (*Folgesätze*)[46], and for this reason it is assumed to be the real ground that makes mathematics indispensable for natural

[44] See F. Beiser, *The Genesis of Neo-Kantianism 1796-1880*, Oxford-New York, Oxford University Press, 2014.

[45] A. Menne, *Drobisch, Moritz Wilhelm*, in *Neue Deutsche Biographie* (NDB), ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, Bd. 4, Berlin, Duncker & Humblot, 1959, p. 127. On the topics we are going to deal with, see L. Kreiser, *Was denken wir, wenn wir denken? Wilhelm Drobischs Beitrag zur Entwicklung der Logik*, in *Moritz Wilhelm Drobisch anlässlich seines 200. Geburtstages*, ed. by U.-F. Haustein, L. Kreiser and G. Wiemers, Stuttgart-Leipzig, Hinzel, 2003, pp. 17-25.

[46] M.W. Drobisch, *Neue Darstellung der Logik nach ihren einfachsten Verhältnisse mit Rücksicht auf Mathematik und Naturwissenschaft*, Hamburg-Leipzig, Voss, 1887[5], §§ 2-3, pp. 3-5 (my translation).

science[47]. Finally, Drobisch states that a «purely synthetic construction of logic in light of the example of mathematics would not be appropriate; on the contrary, it is hardly feasible»[48].

This quote is not paradoxical. Drobisch is referring to Kant's intuition[49]. It is clear, to him, that logic should follow another path, which in Kant's terms would be formal or analytic. In fact, if «thought is in general the bringing together of a manifoldness (*eines Vielen und Mannigfaltigen*) into a unity»[50], the kind of «representations» we encounter here are not those of a subject. Rather, they must regard only the «relationships of what is thought»[51].

Such a logical immanentism is finally typified in the reference to Hegel:

> The concept of the universal is more comprehensive than that of the abstract. In the first place, one may distinguish (by borrowing at least the denomination from Hegel) between *abstract* and *concrete universality*. The former pertains to the kind [*Gattung*], provided that, considered in and for itself, it drops all special differences; the latter to the species [*Art*], provided that it contains in itself the universal of the kind, although the latter is limited through the specific

[47] Ivi, § 7, p. 9.

[48] Ivi, § 3, p. 5.

[49] For a perspicuous definition of the term, see R. Torretti, *Philosophy of Geometry from Riemann to Poincaré*, Dordrecht *et al*., Reidel, 1984, p. 164. Also, it is important to note that Kant had already purported to show universals «*in concreto*» in mathematics. He was equally convinced that concreteness was not empirical, but consisted of an *a priori* exhibition of the content in the constructed concept (I. Kant, *Kritik der reinen Vernunft* (*KrV*), in Id., *Gesammelte Schriften* (*GS*), Bd. 1-22, ed. by the Preußische Akademie der Wissenschaften, Bd. III, pp. 468-471; Engl. trans. by P. Guyer and A.W. Wood, *Critique of Pure Reason*, Cambridge *et al*., Cambridge University Press, 1998, pp. 631-632). In sum, he meant a kind of reasoning based on Euclidean geometry and Newtonian physics (M. Friedman, *Kant and the Exact Sciences*, Cambridge, Harvard University Press, 1992, p. 95), so that we can identify intuition mainly with diagrammatic reasoning.

[50] Drobisch, *Neue Darstellung*, § 4, p. 5.

[51] *Ibidem*.

> difference. But the concept of the concrete universal goes even further. If one relates to the kind neither a *determined* specific difference nor leaves this difference totally *undetermined*, and rather thinks that it is *variable*, such a difference can have the property to represent progressively the specific differences of all of the types of the kind, therefore one can call this the *common concept* [Gesamtbegriff] of the whole series of species. Concrete universality pertains to this concept. Indeed, the particular of *all* species is thought of through the universal of the kind and a series of determined but changeable specific differences. Every mathematical formula, which grasps a determined series of numerical values under itself, possesses this concrete universality[52].

In a nutshell, if we suppose that we can vary the content of a kind, we have universality, determination and the particular when the formula is followed out. Accordingly, Drobisch asserts that «every function represents such a universal law that, in virtue of the successive values that the variables may assume, conceptually grasps at the same time under itself all the single cases for which it applies»[53]. In short, the kind is the equation, and the species are the series of values by which the equation is satisfied[54]. In so doing, Drobisch has generalised Hegel's approach: every function is a concrete universal.

As to Cohen's philosophy of mathematics, the main source is of course his book on the history of calculus, where Hegel surfaces in important places, though not always directly.

To begin with, there are many passages whereby Cohen claims that calculus realises the unity between finiteness and infinity. Here is an example: «This was for us the meaning of the tangent line problem: that in the tangent the concept of curve was defined, the

[52] Ivi, § 19, p. 22.

[53] Ivi, § 19, p. 23.

[54] These arguments can be also found in Lotze's *Logik* (E. Cassirer, *Substance and Function and Einstein's Theory of Relativity*, trans. by W.C. Swabey and M.C. Swabey, New York, Dover Publications, p. 19, pp. 23-24). See on this point: J. Heis, *Ernst Cassirer's* Substanzbegriff und Funktionsbegriff, «Hopos», IV (2), 2014, pp. 241-270.

curve itself is generated (*erzeugt*). *The infinite let the finite emerge from itself* (Das Unendliche lässt das Endliche aus sich entstehen)»[55].

The reference to Hegel is thus important to evaluate Cohen's reworking of Kant's principle of the anticipations of perception, the subject to which Cohen's book is devoted[56]. In short, if Kant aimed to endow the real with an intensive magnitude *a priori* in the second edition of his first critique[57], that is, to understand perception as a differential response to external stimuli, Cohen purports to show that it is rather «reality» (*Realität*) itself to be generated through infinitesimals. Nevertheless, he speaks either of «*Bewusstsein des Denkens*»[58] or of «*wissenschaftliches Bewusstsein*»[59] to disclose a kind of not empirically infected consciousness that shapes reality. And so, he assumed that a transcendental approach should be preserved[60]. But again, a more objectivist stance lurks around the corner: «Infinitesimal number [...] ascribes reality to being in the quality. This actualising meaning of the number comes to the breakthrough in the concept of *function*»[61].

This notwithstanding, the evaluation of Cohen's position with respect to Hegel remains difficult. On the one hand, Cohen praises

[55] H. Cohen, *Das Prinzip der Infinitesimal-Methode und seine Geschichte*, Berlin, Dümmler, 1883, § 45, p. 41 (my translation). See also ivi, § 37, p. 32.

[56] G. Gigliotti, *Avventure e disavventure del trascendentale*, Napoli, Guida, 1989, pp. 50-52; Giovanelli, *Reality and Negation*, pp. 178-198; S. Edgar, *Leibniz's Influence on Hermann Cohen's Interpretation of Kant*, «Kant e-Prints», XVI (2), 2021, pp. 200-230.

[57] *KrV*, pp. 151-158 (285-290).

[58] Cohen, *Das Prinzip*, § 42, p. 37.

[59] Ivi, p. III.

[60] See Cohen's dispute against Fechner and psychological empiricism, ivi, §§ 111-112, pp. 160-162. Also see M. Heidelberger, *Die innere Seite der Natur. Gustav Theodor Fechners wissenschaftlich-philosophische Weltauffassung*, Frankfurt a.M., Klostermann, 1993, pp. 249-258.

[61] Cohen, *Das Princip*, § 45, p. 41.

Hegel's reading about the qualitative character of calculus[62]. On the other, he believes that the attempt at fixating ultimate ratios as limits overlooks the «erzeugende *Moment*» that allows us to shift from the point to the curve, and consequently to set forth the beginning of motion in physics[63]. This may sound a bit paradoxical, considering that Hegel's *Logic* goes from mathematics to physics, from «quantity» to «measure»[64]. However, we have said that Hegel was trying to discard the 'mystic' generative lingo that can be traced back to Newton, who is thus both a positive and a negative reference[65].

Now, if we analyse Cohen's discussion of Kant's «limits» and infinite judgements – Cohen calls them «limiting judgements»[66] –, something peculiar takes place. Hegel's concept of relation was mainly a response to the naïve assumption that variable magnitudes equal and do not equal *0*[67]. For Cohen, limiting judgements are conversely of the kind '*A* is non-*B*', in such a way that a subject is predicated only with a property that defines *B* negatively[68]. If I am not mistaken, we should so figure out that the differential is not yet a curve, but not that it is both a curve and a non-curve. Cohen concludes that

[62] Ivi, § 85, pp. 118-120. See Wolff, *Hegel und Cauchy*, pp. 245-249. By the same token, the emphasis on Leibniz's understanding of quality in mathematics is crucial for Cassirer too. See E. Cassirer, *Leibniz System in seinen wissenschaftlichen Grundlagen*, Marburg, Elwert, 1902.

[63] Cohen, *Das Princip*, § 39, p. 34. On the relevance of mechanics for Cohen's project, see M. Giovanelli, *Hermann Cohen's* Das Princip der Infinitesimal-Methode: *The History of an Unsuccessful Book*, «Studies in History and Philosophy of Science», LVIII, 2016, pp. 9-23, p. 12.

[64] Wolff even pointed out that Hegel was aligning with Kant's *constitutive* use of categories, see Wolff, *Hegel und Cauchy*, p. 262.

[65] Ivi, pp. 250-251.

[66] A review of Cohen's interpretation of infinite judgements is to be found in: H. Pringe, *Infinitesimal Method and Judgement of Origin*, «Kant e-Prints», XVI (2), 2021, pp. 185-199.

[67] This was D'Alembert's objection against Newton, see Wolff, *Hegel und Cauchy*, p. 250.

[68] Cohen, *Das Prinzip*, § 41, pp. 35-37.

«the point of the tangent and the point of the curve cannot be considered as two coincident points anymore; quite the contrary, they are one point considering the generation of the curve»[69].

Once again, it is hard to assess whether this is consistent with Hegel's opinions. Apparently, contradiction is incorporated into mathematics[70] and the unity of the notion of function-relation is preserved; nevertheless, the actualisation of the mathematical infinite seems to involve for Cohen bad infinity: «It would be an advantage to determine at least the infinite concept of kind [*den unendlichen Gattungsbegriff*], whereby the positive next kind cannot be reached»[71].

Technically, this is due to the emphasis put on the differential rather than on the differential quotient. This argument gave rise to a ferocious debate within the Marburg School[72]. As far as Cassirer is concerned, I would say that he sought a way to make mathematics *concrete* without entering the field of physics and Cohen's understanding of calculus. This advancement in the project of scientific idealism is now well recognisable within literature[73], although it is still difficult not to admit that it was initiated when Cohen put «*Erkenntnis*» and «*Geltung*» in the foreground[74].

4. *Cassirer's Interpretation of the Concrete Universality of Functions: The Presence of Hegel in Cassirer's Philosophy of Mathematics*

The debate concerning Cassirer's philosophy of mathematics intensified in the last two decades. Cassirer's deep analysis of the foundations of mathematics grabbed the attention of scholars who

[69] Ivi, § 39, p. 34.

[70] Moretto, *Hegel e la «matematica dell'infinito»*, pp. 183-185.

[71] Cohen, *Das Prinzip*, § 41, p. 35.

[72] Giovanelli, *Hermann Cohen's* Das Princip, pp. 17-21.

[73] H. Pringe, *Cohen's* Logik der reinen Erkenntnis *and Cassirer's* Substanzbegriff und Funktionsbegriff, «Kant Yearbook», XII (1), 2020, pp. 137-168.

[74] See B. Veit, *Hermann Cohens Infinitesimal-Logik*, Dissertation, Bayerische Julius-Maximilians-Universität Würzburg, 2017.

unanimously acknowledged his commitment to early mathematical structuralism[75]. This had important consequences as regards the reform and interpretation of both Kant's intuition and intuitionist methods. Cassirer's structuralism is in fact akin to a «*sui generis* logicism»[76] by which he vindicated the productive nature of mathematical thinking. That is to say, it is possible for Cassirer to amplify our knowledge through mathematics in a purely logical way.

Of course, I am not undermining the import of Cassirer's early dispute with Couturat and Russell and his defence of the synthetic nature of mathematical knowledge[77]. I am however emphasising Cassirer's point of view in the post-Kantian tradition, which can be interpreted precisely in view of Hegel's approach to mathematics.

Classic case studies are Dedekind's «cuts» (*Schnitten*) and Cantor's transfinite arithmetic. A cut justifies the introduction of irrational numbers if it partitions the whole set of rational numbers into two subsects which do not have respectively a higher and an inferior term. Let us call S_1 the set that contains all rational numbers lower than x and S_2 the set that embraces all rational numbers bigger than the same variable. The cut generates an irrational number if and only if no squared element of S_1 equals x, and if and only if no square root of numbers in S_2 is equal to x. By way of example, this is obtained

[75] See F. Biagioli, *Ernst Cassirer's Transcendental Account of Mathematical Reasoning*, «Studies in History and Philosophy of Science», LXXIX, 2020, pp. 30-40.

[76] I borrow the expression from: L. Amaral, *Ernst Cassirer's* sui Generis *Logicism: On the Reception of the Logicist Thesis and its Role in* Substanzbegriff und Funktionsbegriff, «Cognitio-Estudos», XIV (2), 2017, pp. 186-198. However, I also have in mind Cassirer's endorsement of Hilbert's formalistic program and the *fruitfulness* of «implicit definitions». See G. Schiemer, *Cassirer and the Structural Turn in Modern Geometry*, «Journal for the History of Analytical Philosophy», VI (3), pp. 182-212; F. Biagioli, *Cassirer in the Context of the Philosophy of Mathematics*, in *Cassirer in Contexts*, ed. by A. Karalus, P. Parszutowicz, Hamburg, Meiner, 2023, pp. 177-196.

[77] See L. Laino, *Russell and Cassirer as Leibniz's Interpreters: On the Analytic and Synthetic Nature of Mathematical and Physical Knowledge*, «Studia Kantiana», XX (2), 2022, pp. 117-136.

when x is $\sqrt{2}$. Hence, Cassirer infers that the foundation of irrational numbers succeeds

> Within the pure arithmetical field [...]. Seen as an ordinal number, the number means but a 'position': it is thereby a necessary and consequent continuation [*Weiterführung*], everywhere we manage to do it, to fix a position as single in virtue of a determined conceptual prescription, instead of considering a new number as 'given'. Indeed, givenness can mean here, where we move completely in the field of purely ideal settlements [*Setzungen*], nothing but fully logical definiteness [*Bestimmtheit*], unambiguousness of a conceptual operation[78].

Two aspects are striking. First, Cassirer maintains that a purely logical foundation of the concept of number is synthetic for it enables a «continuation» based on the insertion of elements in specific places according to a rule. For this reason, numbers are but «positions in structures»[79]. Second, as a corollary, the kind of definiteness of such positions stems from a conceptual operation. The 1907 essay also contains a clear statement that continuity cannot be defined through visual methods. Continuity is not about the drawing of a line without interruptions on a paper or in the mind; it is simply obtained by providing the *definition* of irrational numbers as limits.

Cassirer radicalises his approach based on Cantor's transfinite arithmetic, to which he devoted a great – but sometimes underrated – interpretive effort. I leave below a couple of excerpts that I judge to be pivotal:

> After considering the completion that the numerical field undergoes 'inwards' through the settlement of irrational numbers, we turn to the not less significant *amplification*

[78] E. Cassirer, *Kant und die moderne Mathematik* (1907), in *Gesammelte Werke* (*GW*), Bd. 9, ed. by M. Simon, Hamburg, Meiner, 2001, p. 49.

[79] J. Heis, *Arithmetic and Number in the Philosophy of Symbolic Forms*, in *The Philosophy of Ernst Cassirer. A Novel Assessment*, ed. by J.T. Friedman and S. Luft, Berlin-München-Boston, De Gruyter, 2015, pp. 123-140.

> [Erweiterung] of its original sphere that occurs because of the introduction of Cantor's transfinite numbers[80].
>
> The 'infinite number' [...] wholly clarifies that the concept of number does not arise from the actual enumeration of whatever given empiric plurality, but it rests on the universal conceptual function by means of which we connect into unity a manifold in virtue of its generating law that we can realise [*vergegenwärtigen*] totally and all at once[81].

In outline, these are comments that follow the introduction of transfinite numbers. Cassirer is mainly focused on ordinal transfinite numbers. While cardinal transfinite numbers refer to the equality of «power» (*Mächtigkeit*) between different sets[82], this new type of number denotes another feature of mathematical entities. By way of example, $\mathbb{Q}$ is dense everywhere if we simply enumerate its elements, and so we are under the influence of what Cantor calls the first «*Erzeugungsprinzip*» of numbers. Nevertheless, this bad infinite is surpassed as soon as we learn to coordinate the series with $\mathbb{N}$. This will alter the disposition of elements in $\mathbb{Q}$ but will give us the idea that we are in the realm of 'countable' sets. Therefore, we may create a type of number ω that identifies the 'order' of this type of series. At this point, we are ready to shape a brand-new «*Erzeugungsprinzip*» which allows us to prove that real numbers are different from what is countable[83], and in general that infinite enumerations correspond to different definite objects.

In Kantian terms, every mathematical creation can be accordingly seen as an amplification of an already existing mathematical structure. It is thus natural for Cassirer to conceive of this process as the *synthetic* generation of concreteness within mathematical concepts. But Kant's original concept might lead us

[80] Cassirer, *Kant und die moderne Mathematik*, p. 56.

[81] Ivi, pp. 59-60.

[82] Ivi, p. 58.

[83] For technical details: Ø. Linnebo, *Philosophy of Mathematics*, Princeton, Princeton University Press, 2017, pp. 58-62; D.F. Wallace, *Everything and More. A Compact History of Infinity*, New York-London, Norton & Co., 2003.

astray. In footnote n. 49, I have already pointed out that intuition was related to diagrammatic proofs. In this case, the construction of concepts is mostly replaced by inferences from symbols. Hence, although Hegel was sceptical about formalism, the true infinite and concrete universality seem to be more appropriate notions rather than intuition to survey Cassirer's case. Besides, scholars have underlined that Hegel's philosophy of mathematics has something to do with Cantor's foundation of transfinite arithmetic[84], and Cassirer is certainly no less committed to Cantor's claims for obvious historical reasons. It is then no surprise that Cassirer seems to uphold the view that we should not confuse the true with the bad infinite:

> The 'material' of enumeration at our disposal is unlimited, for it is not of an empirical but of a logico-conceptual nature. It is not assertions concerning things that are to be collected, but judgments concerning numbers and numerical concepts; thus the 'material,' which is presupposed, is not to be thought of as outwardly given but as arising by free construction. The concept of the transfinite [...] represents the independence of the purely logical import of number from 'enumeration' in the ordinary sense of the word[85].

What I am trying to say is that, through a Kantian wording, Cassirer is hinting at a Hegelian foundation – or what he thinks is Hegelian – for mathematical concepts that makes Hegel's assertions even more radical, and precisely within the field of mathematics and its symbols. Indeed, this is what paradoxically distinguishes Cassirer from Hegel.

Cassirer's standpoint is summarised in the passage from *Substanzbegriff und Funktionsbegriff* to which I referred in the opening of the third section. He writes:

> Modern expositions of logic have attempted to take account of this circumstance by opposing, in accordance with a well-known distinction of Hegel's, the abstract universality of the

[84] Moretto, *Hegel e la «matematica dell'infinito»*, pp. 181-183.

[85] Cassirer, *Substance and Function*, p. 65.

> concept to the concrete universality of the mathematical formula. Abstract universality belongs to the genus in so far as, considered in and for itself, it neglects all specific differences; concrete universality, on the contrary, belongs to the systematic whole [*Gesamtbegriff*] which takes up into itself the peculiarities of all the species and develops them according to a rule[86].

It is interesting that such a lawful development was what Drobisch took from Hegel and that, some years later, Cassirer will define the «universal» as follows:

> The universal of the concept of law contains the particular of the singular case not only, as the species, *under* itself, but veraciously *in itself*: it does not determine in them only a part that can be highlighted at will, but it rather subordinates them in their entirety to the rule of a necessary connection, although also here the particularities of the application [...] are not deducible as such in the manner of the synthetic universal from the form of the law[87].

Apart from technical issues caused by the eradication of arbitrary functions from the general definition of the concept of function[88], one may object that, in that book, Cassirer was contrasting Kant's «analytic universality» from the *Critique of the Power of Judgment* with Hegel's «synthetic universal», in order to side with the former. But from the 1920s, Cassirer's attacks against Hegel are mainly motivated by historical and political grounds[89]. So, it is likely that the definition of analytic universality still couples with that of concrete universality when applied to mathematics. We encounter here nothing but the 'following out' of laws. Hence, we should restate that, according to Cassirer, mathematics is synthetic, but not

[86] Ivi, p. 20.

[87] E. Cassirer, *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit*, Berlin, Bruno Cassirer, 1920, p. 373, fn 1.

[88] Biagioli, *Cassirer in the Context of the Philosophy of Mathematics*, p. 180, fn 9.

[89] See L. Laino, *Dall''universale concreto' all''universale analitico'. La filosofia del 'primo' Cassirer tra Kant e Hegel*, «Giornale di metafisica», XLIV (2), 2022, pp. 523-546.

in Kant's sense[90]. It is for this reason that Hegel and the concept of concrete universality are not a simple sparring partner for him and that, among Cassirer's scholars, there are those who noticed the shift from Kantianism to Hegelianism:

> The Kantian issue of the transcendental schematism of the understanding flows into the Hegelian question of ascribing concreteness to the concept and the universal. Cassirer tacitly evokes this approach on a neo-critical background: arising from functions, the concept amounts to 'concrete universality', in the mathematical field and in other fields as well[91].

However, two divergences are on the verge of being disclosed. First, Cassirer could not underestimate the importance of symbolism in mathematics. Symbols are not merely the vestiges of conceptual determinations but constitute the relations that enliven concepts. Referring to ordinal transfinite numbers, Cassirer maintains that «these are by no means introduced here as mere arbitrary symbols but are signs of conceptual determinations and differences, that are actually given and can be definitely pointed out in the field of infinite groups»[92]. Second, Gigliotti highlighted that there was a difference between the concept of ratio and that of function[93]. While Hegel would see in the former the very realisation of the true mathematical infinite, the neo-Kantians would conversely prefer the

[90] Smart has shown that there is an analytic turn in Cassirer: H. Smart, *Cassirer versus Russell*, «Philosophy of Science», X (3), 1943, pp. 167-175. Indeed, a succinct definition of Cassirer's peculiar understanding of the synthetic can be adapted from Coffa's assessment of Russell's early work: «The synthesis in mathematical and logical knowledge can be produced from concepts alone, without appeal to any kind of intuition» (J.A. Coffa, *The Semantic Tradition from Kant to Carnap*, Cambridge-Melbourne, Cambridge University Press, 1991, p. 46). This recalls Hegel's criticism of Kant's intuition, see A. Moretto, *Filosofia della matematica e della meccanica nel sistema hegeliano*, Padova, il Poligrafo, 2004, pp. 115-122.

[91] L. Lugarini, *Critica della ragione e universo della cultura. Gli orizzonti cassireriani della filosofia trascendentale*, Roma, Edizioni dell'Ateneo, 1983, p. 81.

[92] Cassirer, *Substance and Function*, p. 64.

[93] Gigliotti, *Avventure e disavventure*, pp. 135-138.

latter. So, an alternative seems to surface as to the embedding of infinity in relations and the endless generation of lawfully related values following functions. But we have shown that the notions of relation and function largely overlap in Hegel as well[94]. Findlay clearly stated that «*True infinity is, in short, simply finitude essentially associated with free variability*»[95].

Cassirer accepts this idea when he outplays the psychologistic meaning of enumeration: with transfinite numbers, we derive each mathematical object from a law, so that infinity is both the *process* and the *result* of the correlation one-to-many realised by symbols. This has nothing to do with counting in the ordinary way, i.e., with the «and-so-on» infinity[96]. Furthermore, that Cassirer distanced himself from both Kantianism and neo-Kantianism was clear from a letter sent to Natorp on 30 October 1909. Explaining that Dedekind's cuts «deploy a universal *principle* according to which the totality of all possible cuts appears as an *ordered* totality»[97], which so develops «*direkt*», Cassirer affirms that:

> Since all *mathematical* particulars are in any case nothing but expressions of relations, it follows that every relation which is unambiguous in itself can be expressed and symbolised through a particular (*ein Individuum*). Besides, we may also throw out this aspect of Dedekind's approach without touching its essence and core – as for instance Russell does by replacing clearly and explicitly Dedekind's cuts with the infinite classes through which they are defined[98].

So, in 1909, Cassirer already imbued his philosophy of mathematics with a hint of Hegelianism[99].

[94] Wolff, *Hegel und Cauchy*, pp. 240-241; pp. 254-255.

[95] J.N. Findlay, *Hegel. A Re-Examination*, London-New York, Routledge, 1958, p. 164.

[96] Cassirer, *Substance and Function*, pp. 64-66.

[97] Cited in H. Holzhey, *Cohen und Natorp, Band II: Der Marburger Neukantianismus in Quellen*, Basel-Stuttgart, Schwabe & Co., 1986, p. 379.

[98] Ivi, pp. 379-380.

[99] See also Cassirer's definition of continuity as «continuation of the one and the same law of function» (ivi, p. 382). As to Cassirer's divorce from Cohen, see T.

5. *Concluding Remarks*

For reasons of space, I will limit myself to sum up our achievements with the following table:

Hegel	Topic	Cassirer
Relation-function: the law encompassing a manifoldness (e.g. Spinoza's eccentric circles, ultimate ratios): confutation of bad as the and-so-on infinity	*The true mathematical infinite*	Relation-function: the law encompassing a manifoldness (e.g. Dedekind's cuts, Cantor's transfinite arithmetic); mathematical are not psychological concepts (infinity cannot be confused with counting)
It is realised in biology, only to a certain extent in mathematics	*Concrete universality (bridging the gap between the universal and the particular)*	It is realised in mathematics
Symbolic methods undermine conceptual determinations, although relations are formulae	*Symbolism*	Symbolic methods are the quintessence of the formation of mathematical concepts

I believe that this table clearly shows the affinities and divergences between Hegel and Cassirer. Also, it explains why the influence of the former on the latter should not be understood in the sense that such an impact did compel Cassirer to adhere to Hegel's system. Quite the contrary, it is an instance of how great philosophers rework the claims of the influential thinkers of the past: they draw on their ideas to develop a refined standpoint according to

Mormann & M. Katz, *Infinitesimals as an Issue of Neo-Kantian Philosophy of Science*, «Hopos», III (2), 2013, pp. 236-280, in particular pp. 272-276.

which new scientific theories can be interpreted naturally. In the case of Cassirer, it is the accomplishment of the arithmetisation of analysis that convinced him that concrete universality is eventually realised in mathematics, precisely for the reasons that led Hegel to think that this did not fully occur at his time.