KANT ON CONCRETE UNIVERSALS: AN INQUIRY INTO LOWEST SPECIES

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Abstract. *This paper reconstructs Kant's notion of concrete universals by illustrating his treatment of lowest species. In doing so, I shall oppose the widespread view that Kant rejects the very notion of a lowest species in the following manner. By distinguishing between two kinds of lowest species in Kant's logic, I first show how his dismissal of the latter relays on non-deductive arguments and concerns empirical concepts only. Secondly, I argue that Kant's notion of correspondence between geometrical concepts and the related figures substantiates the idea that he conceived of geometrical notions as lowest species.*

Keywords. *Kant; Lowest Species; Geometrical Concepts; Correspondence; Construction*

1. *Introduction*

Whether they exist or not, universals consist in entities which admit of multiple exemplifications, whereas the term 'concrete' is often used to qualify individuals, that is, entities which cannot be exemplified. 'Concrete universal' thus represents a *contradictio in adjecto*, which is enough to motivate a philosophical inquiry into this subject.

In what follows, I intend to reconstruct Kant's conception of concrete universals. Although the formula 'concrete universal' does not occur in Kant's works, I believe one can rephrase the aim of this contribution by using a Kantian expression which proves to be a synonym of 'concrete universal'. I take «lowest species» to be this Kantian expression and would like the reader to provisionally trust me in this regard. At the end of this enquiry, it will become apparent

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that illuminating Kant's conception of lowest species amounts to reconstructing his conception of concrete universals[1].

According to a widely held view, Kant rejects the very notion of a lowest species[2]. This article opposes such a reading. In particular, I will first show how Kant's arguments for the non-existence of lowest species, whilst most scholars deem them sound, suffer from several flaws and exclusively hold for empirical concepts. Secondly, I will contend that Kant conceives of geometrical concepts in terms of lowest species.

I proceed as follows. In sec. 2 I provide an interpretation of Kant's notion of a lowest species and argue for the need to distinguish between two types of lowest species in his works. In sec. 3 I reassemble Kant's arguments for the non-existence of lowest species and illustrate how they were misread by several commentators. This allows me to substantiate two theses: Kant's rejection of lowest

[1] Though I will justify the synonymity between «lowest species» and 'concrete universals', one might object that other Kantian locutions would prove to be fitter for the task of reconstructing Kant's conception of concrete universals. After all, Kant uses the word 'universal' to qualify numerous notions (experience, knowledge etc.) which might be called 'concrete' in some sense of the word. Yet in the formula 'concrete universal' 'universal' is a name, whereas in 'universal knowledge' and the like it is an adjective. In contexts where 'universal' is a name (e.g., in current metaphysics and medieval philosophy) this word signifies a multiply instantiable property, that is, something very similar to the Kantian term «concept». Of course, none of this excludes the possibility to assess whether Kantian notions expressed by formulas where 'universal' occurs as an adjective may be considered concrete in some sense of the word.

[2] Here are some examples: C. Tolley, *Kant's Conception of Logic*, Doctoral Dissertation, Chicago, University of Chicago, 2007, p. 376; L. Guenova, *Leibniz, Kant and the Doctrine of a Complete Concept*, in *Kant und die Philosophie in weltbürgerlicher Absicht*, ed. by S. Bacin *et al*., Berlin, De Gruyter, 2013, pp. 335-346, p. 344; L. Anderson, *The Poverty of Conceptual Truth. Kant's Analytic/Synthetic Distinction and the Limits of Metaphysics*, Oxford, Oxford University Press, 2015, pp. 67-71; E. Watkins, *Kant on Laws*, Cambridge, Cambridge University Press, 2019, pp. 212-223; D. Sutherland, *Kant's Mathematical World*, Cambridge, Cambridge University Press, 2021, pp. 217, 219, 259-260, 329-330.

species concerns empirical concepts only and rests on highly vulnerable, non-deductive arguments. By taking two Kantian claims at face value – namely: definitions yield lowest species and «only mathematics has definitions» – I then assume the hypothesis that geometrical concepts form lowest species. Sec. 4 and 5 are devoted to verifying the hypothesis by developing a reading of Kant's notion of geometrical construction. On the basis of this reading, I contend that the conditions for conceptual division established by Kant's logic cannot be satisfied as far as constructed geometrical concepts are concerned. Since non-divisible concepts correspond to lowest species, this amounts to confirming my hypothesis. Finally, sec. 6 argues for the synonymity between «lowest species» and 'concrete universal'.

2. *Conceptual Hierarchies and Lowest Species*

This section aims at clarifying the notion of a lowest species. To this end, I will explicate Kant's notion of a concept and seek to understand what it means to employ a concept «*in concreto*» and «*in abstracto*».

A concept is a «universal representation, or a representation of what is common to several objects, hence a representation *insofar as it can be contained in various ones*»[3]. A concept, then, is a property exemplified by or «contained in» several representations which thus form the «extension» of the concept[4]. Every representation belonging to the extension of a concept is, in Kant's terminology, «contained under» that concept and every concept containing representations under itself constitutes a part of their content or intension – it is «contained in» those representations[5]. Since the «representations»

[3] Reference to Kant's works is made by specifying the relevant volume and page number of the *Akademie Ausgabe*: I. Kant, *Gesammelte Schriften,* ed. by the Berlin-Brandenburgischen Akademie der Wissenschaften, Berlin, 1900ff [=*AA*]. The *Kritik der reinen Vernunft* (in *AA* 3,4; Eng. trans. by P. Guyer, *Critique of Pure Reason*, Cambridge, Cambridge University Press, 1998) is referred to by '*KrV*' followed by the first two editions' page numbers. Here: *AA* 9:91.

[4] Ivi:95.

[5] *Ibidem.*

contained under a concept may be intuitions (individuals) or concepts, Kant's logic allows for two types of extensions: the set of a concept's subspecies (hereafter: 'sphere' of the concept) and the set of individuals exemplifying the concept (hereafter: 'extension' of the concept)[6]. Moreover, given a concept C belonging to the sphere of another concept C', C is said to be lower than or subordinate to C', while C' being higher than C. For instance, the set of rational individuals R = {*Kant*, *Michael*, ...} forms the extension of the concept <rational>, while <angel> and <human>, considered jointly, are the subspecies making up the sphere of <rational>. The concept <rational> is exemplified by every individual belonging to R and is contained in the concepts <angel> and <human>, thereby contributing to form their intensions. I will dub the concepts contained in a concept 'marks' or 'properties' of that concept.

In Kant's logic, «abstract» and «concrete» qualify the employment of a concept or the way of regarding a concept[7]. To apply or to consider a concept «*in abstracto*», e.g., <tiger>, amounts to referring it to a higher concept, as is the case with the judgement: 'Tigers are vertebrates'. The same concept is employed «*in concreto*» when referred to a lower concept, e.g., in the judgements: 'Some tigers are Siberian' and 'Every tiger is either Siberian or Sumatran'. Given a concept C, «abstraction» and «determination»[8] designate logical operations to be carried out in order to obtain, respectively, a higher or a lower concept. Any concept obtained by abstraction contains fewer marks and more subspecies than C does, while any concept acquired by determination contains more marks and fewer subspecies than C does. According to the example above, the concept <vertebrate> may be obtained by abstracting from some of the marks contained in <felid> (such as <mammal>, <carnivorous> etc.), is endowed with more subspecies and contains fewer marks than <felid> does. The concept <tiger>, having been acquired by determination

[6] In support of Kant's double notion of extension see L. Anderson, *The Poverty of Conceptual Truth*, pp. 62-63. In what follows, my usage of 'sphere' aligns with Kant's, while my usage of 'extension' does not.

[7] *AA* 9:99.

[8] *Ibidem*.

from <felid>, contains with reference to the latter a higher number of marks and a lower number of subspecies.

Whereas abstraction consists in the subtraction of certain marks from a given concept's content[9], to determine a concept does not mean to add marks to its content. Indeed, the concept <Siberian tiger> is not obtained by adding <Siberian> to <tiger>, but by «dividing» the sphere of <tiger> in accordance with two conditions: the subspecies resulting from the division must be mutually exclusive and, considered jointly, they must exhaust the divided concept's sphere[10]. Suppose <Siberian tiger> and <Sumatran tiger> to result from dividing <tiger> in compliance with such conditions. Then, applying the concept <Siberian tiger> to an individual implies claiming that such an individual is no Sumatran tiger, and every tiger is either Siberian or Sumatran, *tertium non datur*[11]. Here is why any concept obtained by determination, despite necessarily containing more marks than the divided concept, does not result from the addition of those marks. So, although the specification of a concept's content is the outcome of conceptual determination, Kant insists that to determine a concept means to manipulate its sphere, rather than its content[12].

I now turn to the notion of a lowest species. Abstraction and determination admit to be treated with reference to their degree[13]: reaching <vertebrate> by abstraction form the marks contained in <tiger> requires, in comparison with the abstractive procedure carried out to reach <mammal>, a higher level of abstraction – in turn, <tiger> is more determined than <mammal>. Now, Kant allows for a maximum degree of abstraction but seemingly rejects the possibility of a maximum degree of determination. For in proceeding by

[9] *AA* 9:95.

[10] Ivi:146.

[11] Division is just what Plato's *Sophist* called *diairesis*. Empirical concepts such as <tiger> are usually divided by «polytomy» rather than by *diaresis* (9:147). For the moment, this circumstance is negligible.

[12] *Ibidem*. See *AA* 24:925 as well.

[13] *AA* 9:99.

successive abstractions, one reaches a *genus summum*[14], namely, a concept devoid of content which retains every other concept under itself and renders it impossible to perform any further abstraction: as is the case with <something>[15]. Yet proceeding by successive determination toward concepts endowed with increasingly richer contents and ever-narrower spheres does not yield, according to Kant, a thoroughly determined concept or a concept admitting no further determination. In other words, there are no lowest species: «in the series of species and genera there is no lowest concept (*conceptus infimus*) or lowest species, under which no other would be contained»[16]. While it is possible to acquire an absolutely abstract concept – namely, to complete a series of successive abstractions – a complete determination leading to an absolutely concrete concept appears to be impracticable.

However unequivocal the last quotation may be, the arguments concocted by Kant to justify his claim prove to be vulnerable and limit his rejection of lowest species to the specific case of empirical concepts. This I will argue for in the next section.

In order to lay the ground for my analysis of Kant's arguments for the non-existence of lowest species, I would like to highlight an often-neglected aspect concerning his conception of thoroughly determined concepts.

A concept is a «lowest species» (or is «thoroughly determined») if its content admits no further determination; since the determination of a concept's content results from the subdivision of its sphere, a lowest species is a concept which admits no further division. The division of a concept proves to be impossible if the abovementioned division conditions cannot be met[17], which may be the case under two very different circumstances: either the concept to be divided refers to a single individual or it refers to a set of individuals which cannot be assigned to two or more mutually exclusive subspecies which, jointly considered, exhaust the concept's sphere. Since both circumstances entail the impossibility to divide a concept,

[14] Ivi:97.

[15] Ivi:95.

[16] Ivi:97.

[17] Ivi:146.

Kant's text understandably designates both of them by one expression, be this «thoroughly determined concept» or «lowest species». Nevertheless, Kant proves himself to be perfectly aware of the need to distinguish between the circumstances which block conceptual division. He considers indeed both the case of non-divisible concepts applying to single individuals[18] and the case of non-divisible concepts which are endowed with an extension retaining more than one individual[19]. In the former case a lowest species is an individuating description which, similarly to a Leibnizian complete concept, contains all and only the marks exemplified by a certain individual. In the latter case, on the contrary, a lowest species is a universal representation, i.e., a representation referring to several individuals which impede to meet the division conditions. As it appears, the difference must be drawn since neither of the cases logically implies the other. Therefore, one can maintain the non-existence of lowest species if and only if one rules out the possibility of encountering both cases. And any reconstruction of Kant's arguments for the non-existence of lowest species is incomplete unless it accounts for the distinction in question.

3. *Kant's Arguments for the Non-existence of Lowest Species*

I now turn to the arguments whereby Kant seeks to ban lowest species from the Porphyrian tree.

In the Appendix to the Transcendental Dialectic the rejection of thoroughly determined concepts is presented in terms of a heuristic maxim – the «law of specification» – imposed by reason on the understanding:

> reason demands [...] that no species be regarded as in itself the lowest; for since each species is always a concept that contains within itself only what is common to different things, this concept cannot be thoroughly determined, hence it cannot be related to an individual, consequently, it must at every time contain other concepts, i.e., subspecies, under itself[20].

[18] *KrV* B683-84/A655-56.

[19] *AA* 24:259, 911; *AA* 9:97.

[20] *KrV* B 683-84/ A655-56.

The argument aims at proving that the notion of a lowest concept is a *contradictio in adjecto* and runs as follows: a thoroughly determined concept is a non-divisible concept referring to a single individual; yet being endowed with a sphere and/or an extension retaining several individuals is a necessary condition for a representation to be a concept[21] – every species «is always a concept that contains within itself only what is common to different things». Therefore, the formula «lowest concept» is a *contradictio in adjecto*.

If one adheres to Kant's understanding of concepts as universal representations, i.e., as representations endowed with a sphere and/or an extension, the argument proves to be cogent. As I pointed out in sec. 2, however, «lowest concept» (or «thoroughly determined concept») is a vague expression, for it indiscriminately designates:

(C_i) a non-divisible concept referred to a single individual

and

(C_m) a non-divisible concept endowed with an extension retaining more than one individual.

In order to reject lowest concepts, one is required to show the impossibility of both C_i and C_m. Since it considers the case of C_i, but neglects the case of C_m, Kant's argument does not suffice as a proof for the non-existence of lowest species[22].

Despite not dodging the objection, in the *Jäsche Logic* Kant proves to be aware of C_m as a kind of lowest species when he argues for the following claim: the possibility to refer a concept to several individuals directly – viz., without reference to the concept's subspecies – does not entail that such a concept is a lowest species. «For even if we have a concept that we apply immediately to *individuals*»,

[21] *AA* 9:91; *KrV* B94/A69.

[22] Guenova (*Leibniz, Kant and the Doctrine of a Complete Concept*, p. 344) and Anderson (*The Poverty of Conceptual Truth*, pp. 67-71), instead, take Kant's argument to be sound because they neglect distinguishing between C_m and C_i. In support of Kant's overall rejection of lowest species, C. Tolley even jointly quotes the passages where Kant's texts reveal the distinction between C_m and C_i (C. Tolley, *Kant's Conception of Logic*, p. 376). Watkins (cf. *Kant on Laws*, p. 214) and Sutherland (*Kant's Mathematical World*, pp. 260, 330) follow the same lead. For the reasons I outlined at the end of sec. 2, all these accounts are incomplete.

Kant contends, «there can still be specific differences in regard to it, which we either do not note, or which we disregard (*aus der Acht lassen*)»[23]. The passage clearly refers to C_m and points out what follows: directly referring to single tigers by means of <vertebrate> makes up no guarantee of this concept being a lowest species, since one is merely abstracting from the marks which belong to the subspecies contained under <vertebrate>. Moreover, consider a concept which was not obtained by abstracting marks form subordinate concepts, the latter being unknown. One cannot exclude the possibility of discovering new properties exemplified by the members of the concept's extension and since these properties may allow for the satisfaction of the division conditions, one cannot rule out the possibility to further divide the concept. Accordingly, if 18th-century chemists had held <absorbent earths> to be a lowest species, they would have illegitimately prevented themselves from discovering the subspecies <chalky earths> and <muriatic earths>[24].

Kant's remarks lead to the following argument for the non-existence of lowest species. In order to call a concept a lowest species one must prove the impossibility of further dividing that concept. Still, at least for the case of empirical concepts, the possibility of further dividing them rests on the experience involving the individuals which belong to their extension. And since experience is inexhaustible the possibility of dividing an empirical concept remains open. However, as Kant himself admits, the crucial premise of the argument – experience is inexhaustible – is a non-constitutive judgment a priori[25], i.e., a merely «regulative» or heuristic maxim imposed by reason on the understanding. Moreover, the argument exclusively holds for empirical concepts, since only empirical concepts underlie experience-dependent division conditions[26]. In sum, Kant's argument for the non-existence of lowest species construed as C_m is valid for empirical concepts only and is as problematic as the «idea» of experience being inexhaustible.

[23] *AA* 9:97, italics added.

[24] Cf. *KrV* B685/A657.

[25] *Ibidem*.

[26] *AA* 9:147.

An additional Kantian argument against lowest species which merits consideration was reconstructed by Eric Watkins. According to Watkins, Kant conceives of the subordination between concepts as a conditioning relation, namely, he considers any concept C' which is contained under a concept C in terms of a condition of C[27]. Now, as the faculty to envision the complete series of conditions for a given conditioned, reason entrusts the understanding with an illimited search for increasingly specific concepts. Accordingly, «since the unconditioned can never be given through the senses and no empirical concept can be formed that would refer to the unconditioned [...], reason will always be forced to search for more specific concepts; that is, there will be no lowest empirical concept»[28]. Watkins's proposal is viable as far as he assumes Kant's rejection of lowest species to hold for empirical concepts only. Besides being bolstered by the argument outlined above, this assumption is textually corroborated by the entire Appendix to the transcendental dialectic. For the reasons I will set forth, however, ascribing Watkins's solution to Kant turns out to be impossible.

To ascertain the flaw in Watkins's reading, the following feature of the Dialectic of pure reason shall be considered. Every rational syllogism Kant discusses can be traced back to the generic *modus ponens:* «if the conditioned is given, then the whole series of conditions is given; the conditioned is given; therefore, the whole series of conditions is given». Still, in every *Hauptstück* of the Dialectic the terms «conditioned», «given», and «unconditioned» acquire a different meaning and every different meaning yields to a different type of inference from the conditioned to the unconditioned. Now, Watkins's interpretation would prove to be correct if Kant conceived conceptual division as an example of a *generic* form of inferential path from conditions to the unconditioned. Were this the case, Kant could apply the general thesis of the Dialectic – the complete series of conditions is never given[29] – to the case of conceptual division. Yet the

[27] E. Watkins, *Kant on Laws*, p. 219.

[28] Ivi, p. 220.

[29] Watkins's reading entails the assumption of there being such a general thesis in the Dialectic. Although I find this view untenable, I will assume it for the sake of discussion.

following passage illustrates a completely different picture: «For from the sphere of the concept signifying a genus it can no more be seen how far its division will go than it can be seen from space how far division will go in the matter that fills it»[30].

Here Kant employs an analogy between the division of matter and the division of a concept's sphere and indicates, as a reason for rejecting lowest species, the lack of criteria whereby to establish the completeness of both procedures. The passage at stake, accordingly, does not invoke the idea of concepts being treated as some unspecified conditioned entity, as Watkins contends. Nor does Kant subsume the case of conceptual division under a generic kind of progress from conditioned to the unconditioned. Rather, he compares concepts with a specific conditioned entity – matter – and logical division with a specific process – the division of matter. Along with the other entities studied in the Antinomy of Pure Reason, matter distinguishes itself from other types of conditioned entities in virtue of a peculiar feature, namely: it is given in the intuition. Now, this distinctive mark of matter makes Watkins's interpretation implausible. Without the premise «objects of the senses are given as conditioned»[31]– i.e., if conditioned entities were things in themselves – Kant argues, the entire series of matter's conditions would be actually given. But since matter is an appearance, the regress into the series of conditions is not given, but merely «demanded»[32] by a regulative principle «of the greatest possible continuation of experience, in accordance to which no empirical boundary [encountered in the series of conditions] would hold as an absolute boundary»[33]. Given this picture, the problem arises: differently from matter, concepts to be divided do not consist in appearances or «object of the senses» but are intellectually accessible items. Therefore, the complete series of their conditions is actually given and leads to a lowest species.

Accordingly, the conclusions the antinomy chapter draws concerning matter's divisibility cannot be extended to the case of

[30] *KrV* B683/A655.

[31] Ivi B525/A497.

[32] Ivi B527/A499.

[33] Ivi B537/A509.

conceptual determination. This, however, is precisely what Kant is aiming at – yet in a way which substantially differs from Watkins's reading. For Kant does not treat concepts and conceptual determination as examples of, respectively, a generic conditioned entity and a generically conceived progress from conditioned to the unconditioned: he rather considers the specific case of matter with the aim of drawing conclusions concerning the specific case of conceptual division.

While Watkins's reading cannot be rescued, I think one can obtain a further argument against lowest species by taking Kant's analogy between material and conceptual division seriously. The analogy concocted by Kant is rather clear: given a concept C, the spheres of the concepts subordinated to C are to the sphere of C as the space filled by matter is to matter. Kant employs a single binary predicate, '*x* saturates *y*' or '*x* fills *y*', both to refer to the relation between the sphere of the concepts subordinated to C and the sphere of C and to characterize the relation between the space and the matter filling it. Therein lies the argument's first premise[34]. The inference ticket may then be stated as follows: just as one is able to infer the limit of matter's division from consideration concerning the space filled by matter, one can legitimately infer the limit of a concept's division from consideration concerning a concept's sphere.

Now the question should be posed as to what one is able to infer with regard to the limit of matter's divisibility by considering the space matter fills. Kant provides a crystal-clear answer in the *Metaphysical Foundations of Natural Science*:

> the possible physical division of the substance that fills space extends as far as the mathematical divisibility of the space filled by matter. But this mathematical divisibility extends to infinity, and thus so does the physical [divisibility] as well. That is, all matter is divisible to infinity[35].

[34] Kant's analogy explicitly rests on the metaphorical character of the logical terms 'extension' and 'sphere'.

[35] *AA* 4:504.

Therein lies the second premise of the argument. In virtue of the analogy, one can thus conclude: the division of a concept extends to infinity, therefore, there are no lowest concepts.

The analogy Kant employs in the Appendix, be it noted, perfectly conforms with his characterization of analogy as set forth in the *Prolegomena*[36]. Since Kant holds it possible to describe intellectually accessible entities by analogy with sensible entities[37], the argument blocks the objection raised against Watkins, according to which intelligible entities (concepts) cannot be treated as sensible entities (matter).

This I consider to be Kant's most convincing argument for the non-existence of lowest species. Nevertheless, arguments by analogy allow for invalidation, especially if they are conceived of as non-deductive – which is the case in the *Jäsche Logic*[38]. To underline this circumstance is significant, for at the end of the day Kant's arguments prove to be logically sound only for the case of lowest species referred to a single individual (C_i). Insofar as they aim to rule out the possibility of encountering a non-divisible concept endowed with an extension retaining more than one individual (C_m), Kant's proofs suffer indeed from several shortcomings. The argument appealing to the idea of experience being inexhaustible exclusively pertains to empirical concepts and rests on a problematic premise, while no more than a single counterexample is required in order to invalidate the analogical argument. Now, it is quite striking to see Kant himself alluding to the fatal counterexample – provided his characterization of definitions should be taken at face value: «*conceptus rei adequatus in minimis terminis; complete determinatus*»[39]. Hermeneutic charity demands to read «completely determined concept» as referring to C_m, since Kant's arguments successfully reject the existence of C_i.

[36] An analogy is an identity between binary properties (*in casu*: <*x* saturates *y*>) exemplified by the parts of «wholly dissimilar things» (ivi:357), such as matter and concepts.

[37] See F. Chiereghin, *La metafisica come scienza e esperienza del limite*, «Verifiche», XVII, 1988, pp. 81-106.

[38] *AA* 9:132-33.

[39] Ivi:140.

Therefore, Kant is arguing: a definition yields a C_m, viz. a lowest species referred to several individuals.

In light of Kant's theory of definitions, the last quotation acquires the status of a counterexample undermining the argument by analogy. For, according to Kant, neither categories nor empirical concepts admit to be defined; only mathematical concepts do[40]. One may consider the analogical argument as referring to either any type of concept or to empirical concepts only – in the former case the argument is invalid, in the latter its validity is restricted to empirical concepts, just as is the case with Kant's other proofs. But this alternative shall not detain us here, since one can at any rate conclude: every mathematically defined concept is a C_m and Kant's rejection of lowest concepts concerns empirical concepts only[41].

In order to reconstruct Kant's conception of lowest species, then, one is required to dwell on Kant's treatment of mathematical concepts and verify whether they are lowest species. Therein lies the task of the next sections.

4. *Individuals Corresponding to Concepts*

I concluded the last section by justifying an investigation into Kant's conception of mathematical notions in order to reconstruct his views on lowest species. In particular, I proposed the following working hypothesis: every mathematically defined concept is a lowest species. To verify this hypothesis, one needs to understand what a mathematically defined concept amounts to. Since mathematically defined concepts are concepts defined by «construction», I will provide an account of Kant's notion of construction, limiting myself to the case of geometrical construction for the sake of brevity.

«To construct a concept means to exhibit (*darstellen*) a priori the intuition corresponding to it»[42]. How is this definition to be read?

[40] *KrV* B757/A729.

[41] I leave aside the question of whether categories admit of complete determination.

[42] *KrV* B741/A713.

«A priori» qualifies the kind of intuition involved in conceptual construction as an individual entity accessible through the senses and yet independently from experience; whereas «to exhibit», as Kant explains in the third *Critique*[43], means to point at an intuition exemplifying a concept. Insofar as the exemplification of a property by a given individual makes up a relation of correspondence between the two, Kant's reference to an intuition «corresponding» to the concept may sound redundant. Yet Kant provides a clarification which excludes this reading. Considered «as the construction of a concept (of a general representation)», he goes on to explain, the intuition «[must] express in the representation universal validity for all possible intuitions that belong under the same concept»[44]. Now, this peculiar request cannot be satisfied by the exemplification-relation: a sheet of paper exemplifies whiteness, but conveys no information concerning other white individuals, namely, it does not «correspond» to the concept <white>. The intuition involved in the construction of a concept, on the contrary, «corresponds» to the constructed concept because it instantiates properties which can be ascribed to every intuition belonging to the concept's extension. Far from being redundant, then, the notion of correspondence occurring in the definition of construction forms a technical concept. When referring to the relation between the constructed concept and the exhibited intuition by the notion of correspondence, Kant attributes to the exhibited intuition – which would otherwise be a mere example – the status of a paradigmatical example or archetype. Accordingly, *an intuition corresponds to a concept iff it displays the whole extension of the concept, namely, iff the properties it exemplifies apply to every intuition belonging to the concept's extension*[45]. Since the properties instantiated by every

[43] *AA* 5:342.

[44] *KrV* B741/A713.

[45] This does not mean to conceive of geometrical figures as «universalizable images», thereby neglecting Kant's distinction between concepts and intuition. In a somewhat tendentious spirit, Michael Friedman ascribed this reading to Lisa Shabel (M. Friedman, *Kant on Geometry and Spatial Intuition*, «Synthese», CLXXXVI (1), 2012, p. 237, fn 7). Despite reaching conclusions which amount

individual belonging to a concept's extension form the content or intension of the concept[46], the properties exemplified by the figure involved in the construction of a concept form the content of the latter. Therefore, the notion of correspondence may be expressed as follows too: *an intuition corresponds to a concept iff it exemplifies all the properties belonging to the concept's content*. The notion of correspondence admits of further clarification with reference to the following passage: «only mathematics has definitions. For the object (*Gegenstand*) that it thinks it also exhibits a priori in intuition, and this (*dieser*) can surely contain *neither more nor less than the concept*, since through the explanation the concept of the object is originally given, i.e., without the explanation being derived from anywhere else»[47]. After ruling out the possibility to define empirical concepts and categories, here Kant aims at explaining why (only) mathematical concepts are definable.

A definition is acceptable only if the *definiens* contains all and only the marks which form the content or the intension of the concept to be defined[48]. The figure involved in the process of construction, Kant argues, «contain[s] neither more nor less than the concept». 'The figure contains nothing less than the concept' means: the figure exemplifies all the properties belonging to the constructed concept – which amounts to the definition of correspondence

to Shabel's (L. Shabel, *Kant's Philosophy of Mathematics*, in *The Cambridge Companion to Kant and Modern Philosophy*, ed. by P. Guyer, Cambridge, CUP, 2006, pp. 94-128), my reading of construction rests on Kant's notion of correspondence, whereas she neglects the latter. In virtue of this feature, I believe, my account is less vulnerable to the criticisms raised by Friedman, which, by the way, exclusively regard Shabel's early works.

[46] *AA* 9:98.

[47] *KrV* B757/A729-30, trans. modified, italics added. Here Guyer's translation is mistaken: «through the explanation *of the concept the object* is [...] given», whereas Kant writes: «durch die Erklärung *der Begriff von dem Gegenstande* [...] gegeben wurde». Every recent contribution I know quotes the error, though a correct translation was provided, among others, by J.M.D. Meiklejohn (I. Kant, *Critique of Pure Reason*, trans. by J.M.D. Meiklejohn, New York, Collier, 1902, pp. 533-534).

[48] Ivi B755/A 727 n.; *AA* 9:144.

provided above. 'The figure contains nothing more than the concept' conveys an additional, very significant piece of information: the figure exemplifies *only* the properties belonging to the constructed concept. Kant thus contends that the figure involved in the construction of a concept exemplifies *all and only* the properties belonging to the concept's content. But to display all and only the marks forming the intension of a concept is to constitute the *definiens* of that concept. Two conclusions, then, are to be drawn. First of all, the figure involved in the construction of a concept forms the *definiens* of that concept. Secondly, the correspondence between a figure *f* and a concept C may be further clarified as follows: *f* corresponds to C iff the properties *f* exemplifies apply to all and only the intuitions which form C's extension, i.e., iff it exemplifies all and only the marks forming C's content. So, the figure involved in the construction of a concept portrays the concept's extension and content.

In the passage quoted above, moreover, Kant explains why this is the case by ascribing to mathematical definitions the capacity to generate the constructed concept. To construct a figure, he argues, does not mean to 'depict' or 'copy' the content of a previously given concept in the intuition, but rather to identify the concept's content from scratch. The construction of a figure is thus the «birth certificate» of the corresponding geometrical concept.

Since the pre-critical period, finally, Kant takes every intuition belonging to a mathematical concept's extension to correspond to the constructed concept[49].

This reading enables me to verify whether constructed concepts are lowest species. A constructed concept is a concept exemplified a priori by the *corresponding* intuition, in the sense clarified above. A lowest concept is a non-divisible concept endowed with an extension retaining several individuals and a concept is non-divisible if the division conditions cannot be satisfied[50]. *But to satisfy such conditions is manifestly impossible if every individual belonging to the concept's extension exemplifies all and only the properties belonging to the concept*. Now, the truth of this implication rests on the following

[49] See the so-called *Prize-Essay*, *AA* 2:278.

[50] *AA* 9:148.

consideration: if any individual belonging under the concept's extension E exemplifies all and only the properties belonging to the concept, then one cannot identify any subset $S \subset E$ by referring to a property displayed by the members of S only. Put simply, no property is exclusively displayed by two or more individuals individual belonging to E. But such a property is needed in order to carry on a subdivision. Hence, a constructed concept is a lowest species.

This may strike as obvious, for who would dare to claim the existence of different species of circumferences, rectangles, cones and cylinders? Nevertheless, as I will show in the next section, satisfying the correspondence conditions for a figure and a concept is far from obvious. And since the conclusion I have just drawn essentially rests on the notion of correspondence, the hypothesis of geometrical concepts being lowest species still lacks verification. To verify the hypothesis, I have to scrutinize the possibility to satisfy the correspondence conditions. The next section shall be devoted to this task.

5. *Correspondence Conditions*

In the previous section, I provided a reading of Kant's notion of construction: to construct a concept is to generate a figure corresponding to the concept in pure intuition. I then proposed a definition of correspondence and showed how, if a figure corresponds to a concept, then the latter is a lowest species. A figure corresponds to a concept iff it exemplifies all and only the marks contained within the concept. It is now my task to verify whether and how a figure can instantiate all and only the marks of a concept, viz., to see whether and how the correspondence conditions can be satisfied.

To this aim, consider the triangle ABC as drawn on a piece of paper, i.e., as a completely determined triangle: the figure before your eyes is right-angled, has an area of 44 cm^2, and was drawn by pencil. Since Kant takes ABC to correspond to the concept <triangle>, ABC should instantiate the properties exemplified by all and only triangles. Yet an objection arises: not every triangle is endowed with a 44 cm^2 area, not every triangle is right-angled, so the properties ABC exemplifies do not apply to all triangular individuals. Furthermore, the very same properties are instantiated by non-triangular individuals – e.g., a square of side 12 cm. Hence, ABC is far from corresponding to the concept of triangle. The problem obviously arises for any triangular individual one may consider, so that, as Locke had indicated in a similar vein, an individual corresponding to the concept of triangle is a chimera:

> Does it not require some pains and skill to form the general Idea [read: image] of a Triangle [...] for it must be neither Oblique, nor Rectangle, neither Equilateral, Equicrural, nor Scalenon; but all and none of these at once. In effect, it [...] cannot exist; an Idea wherein some parts of several different and inconsistent Ideas are put together[51].

How, then, is one to verify the correspondence between a single triangular figure and the concept of triangle? A passage quoted in the previous section answers the question: for the correspondence between a triangular image and the concept of triangle to be established, the intuition must be considered «as the construction of a concept»[52]. The image ABC, Kant maintains, corresponds to <triangle> under a very precise condition: ABC must be considered «as the construction», i.e., one is required to focus their attention on the procedure whereby the figure was constructed:

> The individual drawn figure is empirical, and nevertheless serves to express the concept without damage to its universality, for in the case of this empirical intuition we have taken only account of the action of constructing the concept, to which (*welchem*) many determinations, e.g., those of the magnitude of the sides and the angles, are entirely indifferent[53].

What it means to consider a figure as resulting from a construction or to focus on the figure's constructive procedure is still in need of clarification, but the objection raised above may now be rejected as follows. For the «indifferent» properties – i.e., the properties the

[51] J. Locke, *An Essay Concerning Human Understanding,* ed. by P.H. Niddich, Oxford, Clarendon Press, 1975, Book IV, Chap. 7, Sec. 9.

[52] *KrV* B741/A713.

[53] Ivi B742/A714.

objection referred to – have no part in the operation whereby any triangle whatsoever is constructed. On closer inspection, the properties one cannot ascribe to all and only triangular coincide with ABC's measurable features – the 44 cm^2 area, the 90° angle, the sides' length. In a seminal paper, Kenneth Manders called such features of geometrical diagrams «exact» and distinguished «exact» from «co-exact» attributes: «co-exact attributes are those conditions which are unaffected by some range of every continuous variation of a specified diagram [...]. Exact attributes are those which, for at least some continuous variation of the diagram, obtain only in isolated cases»[54]. Hence, to treat a single figure as the outcome of the related construction amounts to directing one's attention to the co-exact properties only and the figure ABC, as considered independently of the exact properties, corresponds to the concept of triangle[55].

I must nonetheless specify what it precisely means to consider a geometrical figure as resulting from construction[56]. Kant expresses his views in this regard when he explains how the single figure corresponds to the constructed concept «only as its schema»[57]. This claim harmonizes with the theses Kant sets forth in the schematism chapter. There, after paraphrasing Locke's concern, Kant maintains that mathematical schemata form a necessary and sufficient condition for a single geometrical figure to correspond to the related constructed concept: «No image of a triangle *would* ever be adequate to the concept of it. For it *would* not attain the generality of the concept, which makes this valid for all triangles, right or acute, etc. but *would* always be limited to one part of this sphere»[58].

[54] K. Manders, *The Euclidean Diagram* (1995), in *The Philosophy of Mathematical Practice*, ed. by P. Mancosu, Oxford, Oxford University Press, 2008, pp. 80-133, p. 92.

[55] Be it noted that, considering the last quotation from the *KrV*, <being right-angled> is an exact property. Despite not being mentioned by Kant, the figure's spatial position should be viewed as an exact property too.

[56] For I would be otherwise offering a circular account of correspondence.

[57] *KrV* B742/A714.

[58] Ivi B180/A141, italics added.

Now, the schema of a geometrical concept consists in the rule one has to apply in order to construct the figures falling under the concept's extension[59]. The schema for the concept of cone, for instance, is: 'let a right-angled triangle rotate along the axis of either the minor or the major cathetus'; whereas for the case of the triangle the schema is: 'joint three such segments that the sum of any two segments is always greater than the remaining one'. Accordingly, a single cone corresponds to the concept of cone iff one conceives of it as a triangle rotating along a cathetus.

I now wish to summarize the results obtained so far. The correspondence relation between a single geometrical figure and the related constructed concept consists in the figure's capacity to exemplify the set of properties instantiated by all and only the individuals which fall under the concept's extension (sec. 4). But recognizing the figure-concept correspondence requires one to consider the figure as resulting from the related construction, viz., from the application of a schema. Not the naked figure, but rather the figure in light of its construction procedure according to a schema displays the set of properties instantiated by all and only the individuals which fall under the concept's extension, i.e., corresponds to the constructed concept. Now, any individual falling under a geometrical concept's extension corresponds to that concept. Moreover, the division conditions cannot be satisfied for individuals corresponding to concepts and a non-divisible concept is a lowest species. Hence, every geometrical concept is a lowest species.

Before I conclude, two remarks concerning Kant's correspondence conditions are in order.

In the first place, the act of considering a single figure as the outcome of construction is no abstraction from the exact properties, but consists in the opposite of abstraction, i.e., attention (to the co-exact properties). In considering ABC in light of the schema relative to triangles, one does not abstract from the «indifferent» properties, but rather retraces the figure's genetic process or re-constructs the figure, so to speak. The distinction between abstraction and attention as numerically different cognitive operations which lead to the

[59] See Friedman, *Kant on Geometry*, p. 237.

same result occurs in both Kant's precritical and critical writings and turns out to be crucial precisely insofar as mathematical concepts are concerned. For if co-exact attributes were obtained by comparing several triangles, reflecting on their common features and abstracting from exact properties – viz., in accordance with §6 of the *Logic*, one would be anew besieged by Locke's problem. Moreover, this would imply neglecting Kant's claim that «through the explanation the [mathematical] concept [...] is originally given, i.e., without the explanation being derived from anywhere else»[60].

Secondly, Kant's correspondence conditions seem to encounter a relevant objection. Consider again the right-angled triangle ABC. ABC instantiates the property established by Pythagoras' theorem: given the hypothenuse AC, $AB^2 + BC^2 = AC^2$. The attribute at issue is co-exact and independent form ABC's dimensions, but does not apply to acute and obtuse triangles, so that ABC's correspondence to the concept of triangle would collapse. And if ABC ceased to correspond to the concept of triangle, one would be able to divide the latter in function of the property established by Pythagoras' theorem and conclude geometrical concepts are not lowest species. The objection is spiteful and seems to be bolstered by the idea of there being multiple subspecies of <triangle>[61]. Still, this difficulty may be overcome on the basis of Kant's distinction between constitutive and derivative attributes:

> Some of them [marks] belong to the thing as grounds of other marks of one and the same thing, while others belong only as consequences of other marks. The former are primitive and constitutive marks [...]; the others are called attributes (*consectaria, rationata*) and belong admittedly to the essence of the thing, but only insofar as they must first be

[60] *KrV* B757-58/A729-30.

[61] Since the objection is ineffective as regards the concepts <right-angled triangle>, <equilateral triangle> and the like, my attempt at overcoming it with regard to <triangle> follows the principle of *lectio difficilior*. Moreover, Kant thinks of <being right-angled> in terms of an exact property (see *KrV* B742/A715; n55 of this article), so that ABC, if considered as the result of construction, is far from exemplifying Pythagoras' theorem. But again: *lectio difficilior*!

> derived from its essential points (*Stücken*), as three angles follow from the three sides in the concept of the triangle, for example[62].

According to Kant's distinction, the property established by Pythagoras' theorem is no constitutive attribute, but a derivative one. For ABC exemplifies Pythagoras' theorem only by virtue of a demonstration which rests on the construction of three squares having ABC's catheti and hypotenuse as their sides. Such a construction is absolutely irrelevant to the construction of the concept <triangle> since it finds no mention in the schema pertaining to triangles. Put simply, one is not required to know Pythagoras' theorem in order to construct or define the concept of triangle.

Here is the second significant remark with regard to the correspondence conditions: to focus on ABC's construction process means to bracket not only the exact attributes of the figure, but the derivative co-exact attributes too, viz., the co-exact properties ascribed to ABC by way of demonstration. If considered as the result of construction, then, ABC does not exemplify the property expressed by Pythagoras' theorem and really exhibits «neither more nor less than the concept [of triangle]» to which it thus corresponds.

Moreover, as Kant's example suggests, the distinction between constitutive and derivative attributes proves to be crucial precisely in the case of mathematical concepts. For if Kant conceived of the marks attributed to a geometrical concept by demonstration as constitutive marks of the concept – if he neglected the distinction, then geometrical concepts would not be definable[63]. As is the case with empirical concepts[64], indeed, one would then have to modify the content of geometrical concepts following every demonstration whereby a previously unknown property is ascribed to the related figures. Thanks to this distinction mathematical concepts thus

[62] *AA* 9:60-61.

[63] See A. Ferrarin, *Construction and Mathematical Schematism. Kant on the Exhibition of a Concept in Intuition*, «Kant Studien», LXXXVI (2), 1995, pp. 131-174, pp. 146-147.

[64] *AA* 9:141-142.

admit of definitions, and by appealing to the above-outlined objection to undermine the equivalence between geometrical concepts and lowest species one risks denying the definability of mathematical notions – nothing less than the main tenet of Kant's philosophy of mathematics. That every definable (i.e., mathematical) concept is a lowest species[65] retrospectively comes as no surprise.

The question may arise as to the concept of triangle actually being endowed with subspecies (<acute triangle>, <obtuse triangle> etc.). One cannot answer it in the negative. Yet this should not prevent one from holding <triangle> to be a lowest species, for the correspondence between any triangular individual and the concept of triangle excludes the possibility to satisfy the division conditions for this concept (sec. 4). This circumstance is worthy of consideration because it provides new information regarding Kant's notion of a lowest species: a lowest concept is a non-divisible concept which may nevertheless have subspecies. Although some geometrical concepts (<triangle>, <conic section>) are endowed with subspecies, their status of lowest species is preserved by virtue of the distinction between derivative and constitutive attributes. For in light of this distinction the properties which possibly enable these concepts to undergo division – e.g., the property established by Pythagoras' theorem – prove to be irrelevant to their construction procedure. But if one follows Kant's request to focus their attention solely on the procedure carried out to construct a figure, then the figure will correspond to the related concept. And if the individuals falling under the extension of a concept correspond to it, then the concept forms a lowest species.

The case of lowest species endowed with subspecies gains further credibility in view of the passage from the *Logic Jäsche* I discussed in sec. 3[66]. There Kant illustrates in terms of the following *aut-aut* the reasons why one could erroneously hold a concept to be a lowest species: *either* because one illegitimately contends that any empirical discovery of properties enabling the concept to be divided is impossible *or* because one deliberately abstracts from such properties. Yet as is apparent, neither of the disjuncts applies to the case

[65] Ivi:140.

[66] I.e., *AA* 9:97.

of geometrical concepts. As suggested above, the act of considering a geometrical figure as the outcome of construction does not require one to abstract from exact attributes, but rather to pay attention to (constitutive) co-exact attributes or to reconstruct the figure. The first disjunct, on the other hand, coincides with the premise substantiating the argument I reconstructed above: experience is inexhaustible. Besides being highly problematic (see sec. 3), this premise is meaningful with reference to empirical concepts only and the mere question of whether it holds for geometrical concepts rests on a category mistake. For despite any bias for the intuitionistic spirit of Kant's philosophy of mathematics, it would indeed be absurd to debate over 'mathematical experience' being exhaustible or not. Mathematics evolves a priori and Pythagoras 'discovered' his theorem in a completely metaphorical sense of the term[67].

6. *Concrete Universals*

I have shown how Kant's stance toward lowest species cannot be reduced to an attempt at showing their inconsistency. In light of «lowest species» being endowed with two meanings (sec. 2), Kant's arguments for the non-existence of lowest species prove to be vulnerable and their validity exclusively pertains to empirical concepts (sec. 3). Especially, I think the reasons for conceiving of geometrical concepts in terms of lowest species rest on a firm ground (sec. 4 and 5). This outcome, after all, was suggested by Kant's claims: definitions yield lowest species and mathematics only has definitions.

[67] One last objection may be risen by those who refused to conceive of a figure's spatial position as an exact property (see fn 55, *supra*): given two perfectly similar squares which occupy different positions in space, one is able to single out the subspecies <square occupying the position *x*> and <square occupying the position *y*>; therefore, <square> makes up no lowest concept. In this case, however, one would be pointing at two subspecies whose extensions contain one individual only, i.e., two lowest concepts of the kind C_i. But Kant rejects the existence of such concepts (sec. 3). A stronger answer to this objection would have to draw on the *Amphibolie-Kapitel*, which I must neglect for reasons of space.

Does this journey coincide with a reconstruction of Kant's conception of concrete universals? Sec. 2 and the conclusions of sec. 4 and 5 allow for answering in the affirmative: «lowest concepts» and 'concrete universals' are synonyms. Insofar as it refers to entities which admit of multiple exemplification, Kant's term «concept» shares an essential feature with today's (or 12th-century's) usage of 'universals'. Moreover, what follows illustrates the synonymity of «lowest» and 'concrete'.

Kant discourages readers from calling a concept «concrete» or «abstract». Since the same concept may be conceived of as either concrete or abstract with reference to a higher or a lower concept respectively, the parlance of «abstract» and «concrete» concepts is relative. That is, no concept is per se «concrete» or «abstract». Instead, using such expressions to designate a concept's use or the way of regarding a concept is more appropriate. For using of a concept C in a judgement involves a comparison with a second concept C' with reference to which C is considered to be lower or higher (cf. sec. 2). Nevertheless, there is at least one notable exception, namely, the *genus summum*: since the concept of <something> is the highest one, it fairly deserves to be called «abstract» per se. According to the conclusions of this contribution (sec. 4 and 5), lowest species form the second exception, namely, they are the only concepts which deserve to be called «concrete». Therefore, «lowest» and 'concrete' are synonyms as well.

One last remark is in order. The conditions under which a figure corresponds to a concept reveal the etymological, quasi-Hegelian meaning of the Kantian term «concrete». For geometry considers concepts «*in concreto*»[68]not merely by referring them to individuals, but mainly because it must conceive of the related figures as resulting from construction. To consider geometrical figures in this manner amounts to recognizing them as entities which *have grown up to become what they are*. This rather inelegant formulation, in turn, aptly conveys the meaning of the Latin term *concretus*.

[68] *KrV* B743/A715.